Western University

Power Transmission Two-Stage Speed Reducer (Gearbox)

Phase 2 Report: Final Design

Gearbox Group 7

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MSE 3380B

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Scope

The objective of this project is to successfully design a two-stage reduction gearbox that will take the input torque and speed of a chosen motor and then step down the speed such that the output shaft will achieve a speed of 170-180 RPM with an associated power of 25 Horsepower. This will require the selection of an AC induction motor and using its datasheet, find the required gear ratio, gear size, material, and the intermediate shaft of the gearbox. Once the values have been calculated, the gearbox will then be designed and assembled in SolidWorks to provide detailed drawings and measurements. The gears utilized must be of a spur design.

Data

The first decision that was made regarding the design of the gearbox was the motor that would be attached to it. The chosen motor was the 0312ES3EOW326T-W22 AC Induction pump motor. The motor has a rated nominal speed of 1125 RPM and an operating horsepower range of 3-100 HP. Assuming the gearbox will have losses between 2 and 4%, standard for a two-stage reduction gearbox, an efficiency of 97% from input to output will be used. Using this efficiency ratio, it will be operating at 26 HP with an associated torque of 121.282 $lb \cdot ft$ (164.5721 $N \cdot m$).

Using those values as the input values to our gearbox, the objective is to design the gears and shaft to translate that input into an output shaft speed of 175 RPM, with a delivered power approximately equal to 25 HP.

The gearbox must be designed with enough durability to withstand occasional moderate shocks with a size of 1 meter in length and width and 1.4 meters in height. The gears and bearings must be able to run for more than 12000 hours while assuming the shaft has an infinite life span. With all these considerations the gearbox must be designed to effectively meet its objective with a safety factor between 1.5 and 2 to meet industry standards whilst remaining fully enclosed.

Concept

As can be seen below in **Figure 1** the design follows a relatively simple gearbox system.

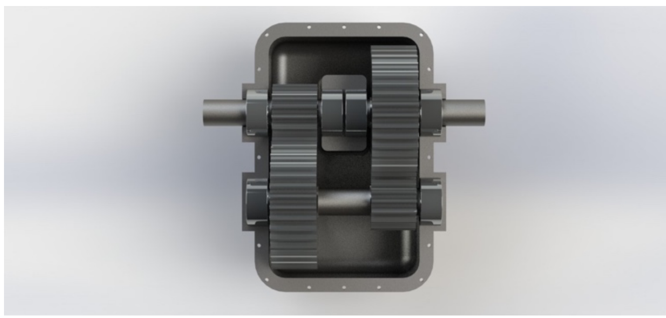


Figure 1 - Open Top View of the Assembled Gearbox

We utilized an input shaft with one gear mounted to it which overlapped with the teeth from a parallel gear on the intermediate shaft. The intermediate shaft would be turned in relation to the input shaft when through these gears the motor is attached to the input shaft and turned on. As the intermediate shaft is turned, it turns a third gear on the opposite end of the shaft at the same speed, which will overlap with the final gear mounted to the output shaft of the gearbox.

As can be seen in **Figure 2** Gears 1 and 2 overlap. Gear 1 is a relatively smaller gear with 15 teeth compared to the larger gear which possesses 38 teeth for a gear ratio of 2.533 between gears, and a total gear ratio of 6.4 from input to output. The calculation of this gear ratio and minimum teeth value can be found below in the report's **Analysis** section. Gears 3 and 4 are identical gears to gears 1 and 2 respectively.

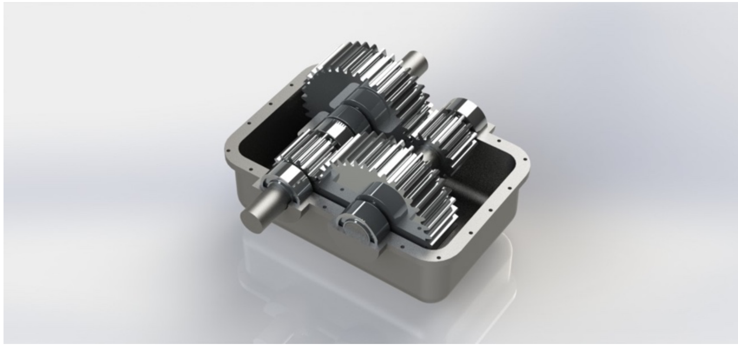


Figure 2- Isometric View of the Open Gearbox

The purpose of using a smaller gear to drive a larger gear is that it forces the speed to be decreased due to the smaller gear being required to make so many more revolutions to equate to the same distance travelled in one revolution by the larger gear. The sizing of the gear was determined with respect to the max sizing of the gearbox and the room we allotted for clearance of the gears and bearings in the box can also be found broken down below, in the **Analysis** section.

Phase 1 Summary

In the ‘Phase 1 Report: Initial Design’ we conducted a thorough analysis to determine the gear and pinion dimensions as well as the speed, torques, and forces associated with them. The results are summarized as follows:

Gear & Pinion Dimensions		
Dimension	**Pinions (2 & 4)**	**Gears (3 & 5)**
Number of Teeth (N)	15 teeth	38 teeth
Diametral Pitch (P)	2 teeth/in	2 teeth/in
Module (m)	12 mm	12 mm
Diameter (d)	0.1905 m	0.4826 m
Face Width (F)	0.1595 m	0.1595 m

Table 1 - Gear and Pinion Dimensions

We found the rotational speed of each gear and pinion, the torques caused by them, and the forces exerted on the gears and pinions. The torque for Pinion '2' is the torque along the input shaft and the torque for Gear '5' is the torque along the output shaft. The speed and torque for Gear '3' and Pinion '4' are equal as they are both securely fastened to the intermediate shaft using keys and retaining clips. The forces acting on them, however, differ due to their dissimilar diameters.

Speed, Torque, and Force				
Measurement	**Pinion '2'**	**Gear '3'**	**Pinion '4'**	**Gear '5'**
Speed (ω)	1125 rpm	444.08 rpm	444.08 rpm	175.29 rpm
Torque (T)	164.572 Nm	416.916 Nm	416.916 Nm	1056.187 Nm
Force (F)	1727.79 N	1727.79 N	4377.07 N	4377.07 N

Table 2 – Speed Torque and Force of Each Gear

Analysis

Once again, the analysis conducted below is with reference to **Figure _** where the corresponding gear and pinion numbering is used throughout.

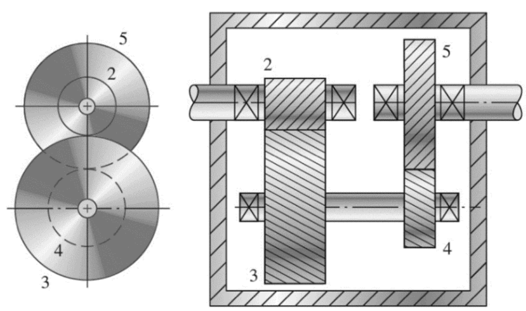


Figure 3 - Gear and Pinion Numbering from Textbook [1]

Problem Specification

This gearbox's objective is to take the input torque and speed from our selected AC induction motor and convert it into a slower predetermined speed with an associated output power through gears. Assuming that our gearbox will use a standard 97% efficiency ratio we need to achieve an output power of 25 HP with an output speed of 170-180 RPM. The gearbox must fit within a set size constraint of 1 meter by 1 meter, with 1.4 meters allowed as the height of the box. Knowing this we will use our motors rated speed of 1125 RPM with an associated torque of 164.572, these values are coming from **Table 2** above.

Gear Specification Cont.

Using the tabulated data provided above, we can find the pitch-line velocities and transmitted loads:

$$V_{23} = \frac{\pi d_2 \omega_2}{12} = \frac{\pi(7.5)(1125)}{12} = 2208.93\ ft/min \qquad \dots(1)$$

$$V_{45} = \frac{\pi d_5 \omega_5}{12} = \frac{\pi(19)(175.29)}{12} = 871.95\ ft/min$$

$$W_{23}^t = 33\,000 \frac{H}{V_{23}} = 33\,000 \frac{25}{2208.93} = 373.48\ lbf \qquad \dots(2)$$

$$W_{45}^t = 33\,000 \frac{H}{V_{45}} = 33\,000 \frac{25}{871.95} = 946.16\ lbf$$

Considering Pinion ‘4’ is the smallest gear, transmitting the largest load, it will likely be a critical component. Therefore, we will start with wear by contact stress, as it is often the limiting factor.

Pinion ‘4’ Wear

Start by finding Geometry Factor ‘I’ where m_N = 1 for external spur gears and m_G must be calculated:

$$m_G = \frac{0.1905}{0.4826} = 2.53 \qquad \text{…(3)}$$

Then, solving for I:

$$I = \frac{cos\phi_t sin\phi_t}{2m_N}\frac{m_G}{m_G+1} = \frac{cos20°\ sin20°}{2(1)}\frac{2.53}{2.53+1} = 0.1152 \qquad \text{…(4)}$$

For K_v, we are able to assume $Q_v = 7$. This delivers values of B = 0.731 and A = 65.1

$$K_v = \left(\frac{A+\sqrt{V_{45}}}{A}\right)^B = \left(\frac{65.1+\sqrt{871.95}}{65.1}\right)^{0.731} = 1.314 \qquad \text{…(5)}$$

The face width F is typically 3 to 5 times the circular pitch, where the circular pitch is $\frac{\pi}{P}$.

$$F = 4\left(\frac{\pi}{P}\right) = 4\left(\frac{\pi}{2}\right) = 6.2832\ in = 0.1595\ m \qquad \text{…(6)}$$

For K_m, we must find appropriate C values:

$$C_{pf} = \frac{F}{10d_p} - 0.0375 + 0.0125F = 0.125 \qquad \text{…(7)}$$

$$C_{mc} = 1\ uncrowned\ teeth$$

$$C_{pm} = 1\ straddle\ mounted$$

$$C_{ma} = A + BF + CF^2 = 0.223\ commercial\ enclosed\ unit$$

$$C_e = 1$$

Therefore,

$$K_m = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right) = 1 + (1)(0.125)(1) + (0.223)(1)) = 1.347 \qquad \text{…(8)}$$

Based on pinion material of Nodular Iron and gear material of Cast Iron we get a value of 2020 for C_p. In our case, $K_o = K_s = C_f = 1$.

So,

$$\sigma_c = C_p \sqrt{\frac{W_{45}^t K_v K_m}{d_4 F I}} = (2020)\sqrt{\frac{(946.16)(1.314)(1.347)}{(7.5)(6.283)(0.115)}} = 35487.8\,psi \qquad \text{…(9)}$$

We are now able to find the number of cycles for the specified life of 12 000h.

$$L_4 = (12000\,h)\left(60\frac{min}{h}\right)\left(444.08\frac{rev}{min}\right) = 3.20 \times 10^8\,rev \qquad \text{…(10)}$$

As the magnitude of L is to the order of 10^8,

$$Z_N = 0.9$$

And assuming,

$$K_R = K_T = C_H = 1$$

We can now find the contact strength for a design factor of 2,

$$S_c = \frac{S_H \sigma_c}{Z_N} = \frac{(2)(35487.8)}{0.9} = 78861.76\,psi \qquad \text{…(11)}$$

A material must now be chosen that has a contact strength greater than the calculated S_c value. According to table 14-7 in Shigley's[1], ASTM A536 ductile (nodular) iron Grade 80-55-06 quenched and tempered will meet our requirements. It has a gear contact strength averaging from 77 000-92 000 psi. So, we will take the median of this range to find:

$$S_c = 84500\,psi$$

Now, we can find the gear contact safety factor for Pinion '4':

$$n_c = \frac{\sigma_{c,all}}{\sigma_c} = \frac{S_c Z_N}{\sigma_c} = \frac{(84500)(0.9)}{35487.8} = 2.14 \qquad \text{…(12)}$$

Pinion '4' Bending

From Figure 14-6 in Shigley's[1], we can find the Geometry Factor 'J' based on the number of teeth that Pinion '4'has.

$$J = 0.25$$

We can now determine the bending stress assuming $K_B = 1$:

$$\sigma = W_t K_v \frac{P_d}{F}\frac{K_m}{J} = (946.16)(1.1.314)\frac{2}{6.283}\frac{1.347}{0.25} = 2133.64\, psi \qquad \ldots(13)$$

As the magnitude of L is to the order of 10^8,

$$Y_N = 0.9$$

For ASTM A536 ductile (nodular) iron Grade 80-55-06, the median value for S_t = 27 500 psi.

Therefore, we can now determine:

$$\sigma_{all} = S_t Y_N = (27500)(0.9) = 24750\, psi \qquad \ldots(14)$$

The associated bending factor of safety for Pinion '4' is then:

$$n = \frac{\sigma_{all}}{\sigma} = \frac{24750}{2133.64} = 11.60 \qquad \ldots(15)$$

For the remaining gears and pinions, we will only demonstrate an abbreviated calculation process as the detailed calculations are shown above.

Gear '5' Bending and Wear

We can find the contact and bending stresses for Gear '5' by conducting the same analysis:

$$\sigma_c = C_p \sqrt{\frac{W_{45}^t K_v K_m}{d_5 F I}} = (2020)\sqrt{\frac{(946.16)(1.314)(1.297)}{(19)(6.283)(0.115)}} = 21872.75\, psi$$

$$\sigma = W_t K_v \frac{P_d}{F}\frac{K_m}{J} = (946.16)(1.314)\frac{2}{6.283}\frac{1.297}{0.38} = 1350.89\, psi$$

Choose ASTM A48 Gray Cast Iron as cast; S_c = 55 000 psi and S_t = 5000 psi

$$n_c = \frac{\sigma_{c,all}}{\sigma_c} = \frac{S_c Z_N}{\sigma_c} = \frac{(55000)(0.9)}{21872.75} = 2.26$$

$$n = \frac{\sigma_{all}}{\sigma} = \frac{5000}{1350.89} = 3.33$$

Pinion '2' Bending and Wear

We can find the contact and bending stresses for Pinion '2' by conducting the same analysis:

$$\sigma_c = C_p \sqrt{\frac{W_{23}^t K_v K_m}{d_2 F I}} = (1960)\sqrt{\frac{(373.48)(1.488)(1.347)}{(7.5)(6.283)(0.115)}} = 23016.05\ psi$$

$$\sigma = W_t K_v \frac{P_d}{F}\frac{K_m}{J} = (373.48)(1.488)\frac{2}{6.283}\frac{1.347}{0.25} = 953.27\ psi$$

Choose ASTM A48 Gray Cast Iron as cast; S_c = 55 000 psi and S_t = 5000 psi

$$n_c = \frac{\sigma_{c,all}}{\sigma_c} = \frac{S_c Z_N}{\sigma_c} = \frac{(55000)(0.9)}{23016.05} = 2.15$$

$$n = \frac{\sigma_{all}}{\sigma} = \frac{5000}{953.27} = 4.72$$

Gear '3' Bending and Wear

We can find the contact and bending stresses for Gear '3' by conducting the same analysis:

$$\sigma_c = C_p \sqrt{\frac{W_{23}^t K_v K_m}{d_3 F I}} = (1960)\sqrt{\frac{(373.48)(1.488)(1.297)}{(19)(6.283)(0.115)}} = 14185.85\ psi$$

$$\sigma = W_t K_v \frac{P_d}{F}\frac{K_m}{J} = (373.48)(1.488)\frac{2}{6.283}\frac{1.297}{0.38} = 603.55\ psi$$

Choose ASTM A48 Gray Cast Iron as cast; S_c = 55 000 psi and S_t = 5000 psi

$$n_c = \frac{\sigma_{c,all}}{\sigma_c} = \frac{S_c Z_N}{\sigma_c} = \frac{(55000)(0.9)}{14185.85} = 3.49$$

$$n = \frac{\sigma_{all}}{\sigma} = \frac{5000}{603.55} = 7.46$$

Therefore, all of the contact and bending safety factors meet the design goal of 1.5.

Shaft Layout

The general layout and of the shafts, gears, pinions, and bearings are shown in the figure below.

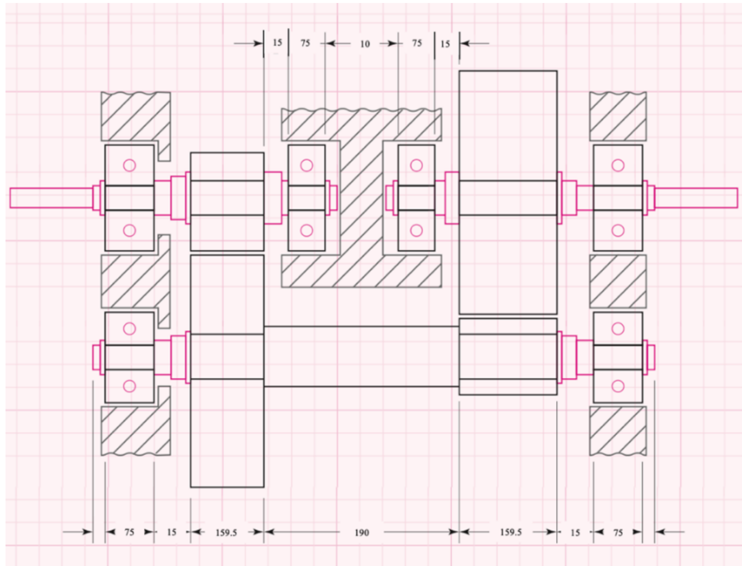


Figure 4 - Shaft Layout

Force Analysis

Free Body Diagram of the intermediate shaft:

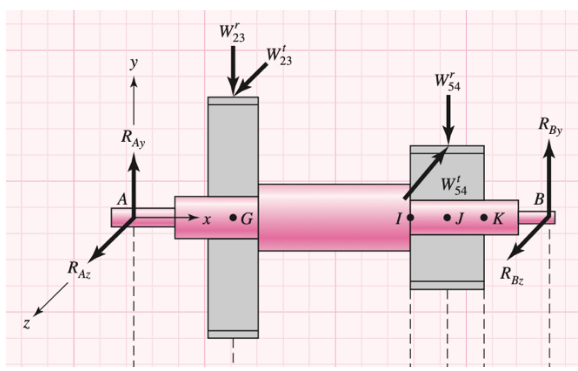


Figure 5 - Free Body Diagram of Shaft

The radial and tangential forces transmitted through the gears to the shaft were determined as follows.

$W^t_{23} = 1727$ N $\qquad$ $W^t_{54} = 4377$ N

$W^r_{23} = 628.6$ N $\qquad$ $W^r_{54} = 1593$ N

Through free-body diagram analysis, we determined the reaction forces at the bearings to be:

$R_{Az} = -412.25$ N $\qquad$ $R_{Bz} = 3062$ N

$R_{Ay} = 836$ N $\qquad$ $R_{By} = 1385$ N

The torque in the shaft between the gears was determined to be:

$$T = W_{23}^{t}(d_3/2) = (1727)(0.4826/2) = 416725\ N \cdot mm \qquad \ldots(16)$$

Torque Diagram

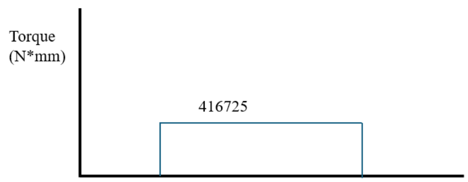


Figure 6 - Torque Diagram of Shaft

Shear and Bending Moment Diagrams

X-Z Plane

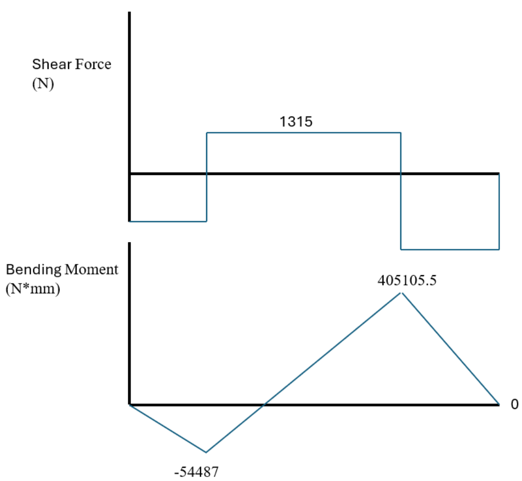


Figure 7 - Shear and Bending Moment Diagrams in X-Z Plane

X-Y Plane

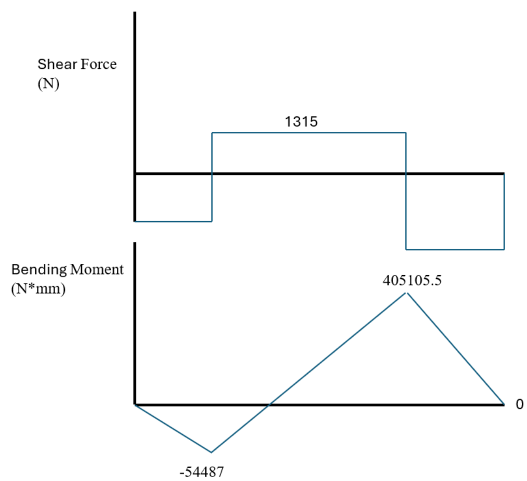


Figure 8 - Shear and Bending Moment Diagrams in X-Z Plane

Shaft Material Selection

A material needed to be chosen for the shaft to find its properties and maximum forces. The material needed to be strong enough to withstand the maximum forces it will endure during operation. A safety factor of two was chosen for the shaft which meant further than that, it had to be able to endure double the maximum force. For this AISI 1020 CD steel was chosen as the shaft material. It offers a Tensile strength of 470 MPa and a yield strength of 390 MPa. These values were taken from Shigley's Mechanical engineering design textbook [1] on page 1056. These values proved to be high enough to withstand and meet the requirements put before it.

Design For Stress

After determining moments and torques as well as deciding on a material for the shaft, we can proceed with the design for the stress procedure. We will show the procedure for location 'I' on the shaft as it is one of the critical points.

At I, M_a = 343 393 N•mm, T_m = 416725 N•mm, $M_m = T_a = 0$. We will estimate $K_t = K_f = 1.7$ and $K_{ts} = K_{fs} = 1.5$ based on a generous fillet radius.

Now, to find S_e,

$$k_a = aS_{ut}^b = 0.815 \qquad \ldots(17)$$

Guess $k_b = 0.9$. Also, $k_c = k_d = k_e = 1$

$$S_e = k_a k_b \frac{S_{ut}}{2} = (0.815)(0.9)\left(\frac{470}{2}\right) = 172.33\ MPa \qquad \ldots(18)$$

For the first estimate of the small diameter at the shoulder point I, use the DE-Goodman criterion.

$$d = \left\{\frac{16n}{\pi}\left(\frac{2(K_f M_a)}{S_e} + \frac{[3(K_{fs}T_m)^2]^{\frac{1}{2}}}{S_{ut}}\right)\right\}^{1/3} = 45.22\ mm \qquad \ldots(19)$$

$$\frac{D}{d} = 1.2\ \rightarrow D = 54.265 \qquad \ldots(20)$$

Assume fillet radius r = d/10 = 4.52 mm

K_t = 1.6, q = 0.83

K_f = 1.498

K_{ts} = 1.35, q_s = 0.86

K_{fs} = 1.301

k_a = 0.815 (no change)

k_b = 0.827

Therefore, the corrected S_e is:

$$S_e = (0.815)(0.827)(0.5)(470) = 158.256\ MPa$$

$$\sigma_a' = \frac{32K_f M_a}{\pi d^3} = 66283\ MPa \quad \text{...(21)}$$

$$\sigma_m' = \left[3\left(\frac{16K_{fs}T_m}{\pi d^3}\right)^2\right]^{\frac{1}{2}} = 51717\ MPa \quad \text{...(22)}$$

Using Goodman criterion:

$$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} \rightarrow n_f = 2.14 \quad \text{...(23)}$$

Check yielding:

$$n_y = \frac{S_y}{\sigma_{max}'} > \frac{S_y}{\sigma_a' + \sigma_m'} = 3.6 \quad \text{...(24)}$$

Ultimately, these values meet the goal for the design factor of 1.5.

We also conducted similar analyses on other critical locations. Those results are summarized as follows:

Shaft Location	**Factor of Safety (n_f)**
I	2.14
Keyway to Right of I	1.60
K	1.63
M	1.52

Thus, location M on the shaft if the most critical location, however, it still meets the design requirements.

Deflection Check

Once the shaft diameters and material were calculated, a shaft was designed in SolidWorks to test for deflection using Finite Element Analysis (FEA). With the designed shaft, it was treated as a beam and had one end treated as fixed geometry to make it so it cannot move to get maximum displacement. Then the maximum force was applied to the shaft and the resultant displacement was mapped. This was repeated to find the deflection of the shaft in the X-Z plane and the X-Y plane. The results of which can be seen in **Appendix A5 and A6** at the end of the report**.** The maximum displacements were 0.003008mm and 0.001834mm respectively. The angle of the slope was around 0.00004 radians, which below the maximum bearing rating.

Bearing Selection

Using the shaft speed of 444 RPM and a lifetime of 12000 hours, we get a design life of 3.2×10^8 revolutions. Using the formula for radial load we can see that on Bearing B there is a 29901 N load. Since this load is large in magnitude it would be a good decision to select a cylindrical roller bearing, as opposed to a ball bearing which has a smaller catalogue load rating. The NJ 306 ECJ cylindrical roller bearing was selected because of its size and catalogue load rating of 58.5kN, which suits our design specifications. For Bearing A the calculated radial load was 8294 N, which a ball bearing would work, but for part simplicity we decided that having both bearings the same type would be more beneficial. So, the same bearing was chosen for A.

$$F_{RB} = 3360\left[\frac{3.2\times10^8/10^6}{0.02+(4.459-0.02)(1-0.99)^{1/1.483}}\right]^{1/3} = 29901\ N \qquad \ldots(25)$$

To find the bearing life using the selected bearing 'A' or 'B', we used the equations below, where a = 10/3 for cylindrical roller bearings. Using the catalog load and lifetime for the selected bearing, we found the bearing lifetime of B to be around 112,000 hours, which far exceeds the minimum requirement of 12000 hours.

$$F_r \times L_r^{\frac{1}{a}} = F_d \times L_d^{\frac{1}{a}} \quad (26) \qquad\qquad L_{Hours} = L_d/(444 * 60) \quad (27)$$

Conclusions

During the first phase of the report, we calculated the required gear ratios, gear sizes and the required input conditions to meet the problem specifications. From there, the second phase required us to determine the stress and bending forces applied to the gearbox's shaft. Along with that we had to calculate the contact stresses for the gears and determine the proper material and safety specifications for both the gears and the shaft. Overall, we believe we were successful in this endeavor. We successfully found the limiting gear and the maximum contact stress it can endure before failing and with a minimum safety factor of 2, we were able to choose a material which would meet these parameters. We were able to calculate and derive the internal forces of the shaft and construct the associated force diagrams. We successfully found and chose AISI 1020 CD Steel as the material for the shaft to endure its maximum forces within our chosen safety factor. We checked our total shaft deflection as well through the use of Finite Element Analysis in SolidWorks. Our bearing was able to be designed to withstand the maximum forces and have a proper life cycle to meet the assigned life span in the problem specifications. These were the tasks we set out to complete, and we were able to complete them with a reasonable degree of safety allowing us to classify our design as viable and successful.

Team

Ryan McCuaig – Gear Calculations and Writing and Editing the Report.

David Dent – Assisted with the Gear Calculations, Report writing and editing, and force diagrams.

Thorben Wennemer – SolidWorks Modeling and drawings of the gearbox and Assisted in Report writing.

Nick Mckim –Shaft and Bearing Calculations, minor SolidWorks Assistance and assisted in report editing and writing.

References

[1] R. G. Budynas and J. K. Nisbett, *Shigley's Mechanical Engineering Design*. New York, NY: McGraw Hill LLC, 2024.

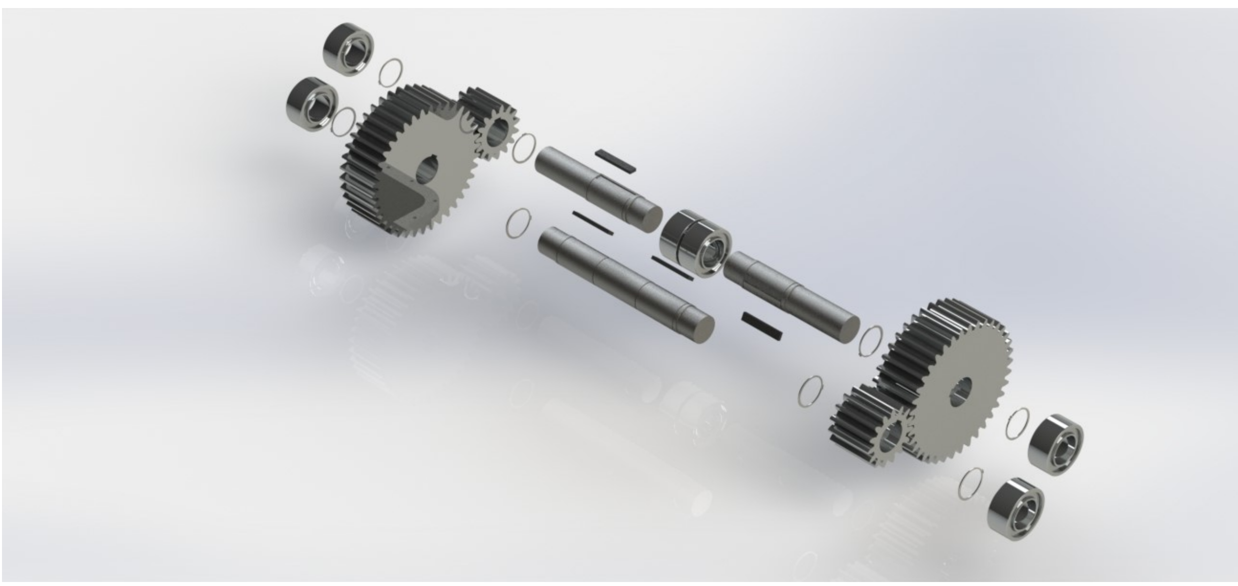
[2] Shopping, https://www.factorymation.com/03012ES3EOW326T-W22 (accessed Mar. 13, 2024).

Appendices

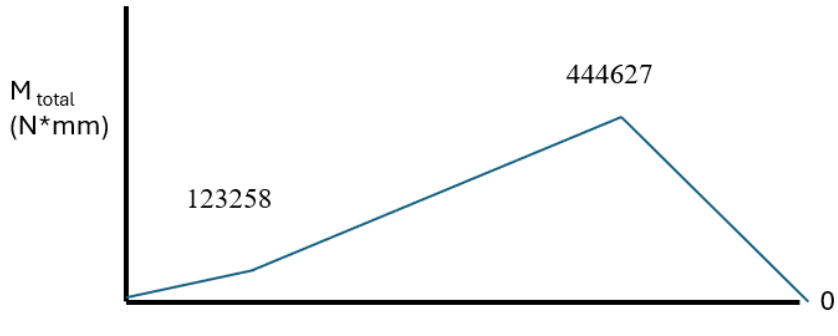
Phase	Three Phase
Frame	324/6T
Output	30 HP
Poles	06
Frequency	60 Hz
Rated speed	1125 rpm
Slip	6.25 %
Full Voltage - No Cables	208-230/460 V
Rated Current	82.0-74.2/37.1 A
LR Amperes	476-430/215 A
LRC	5.8x(Code G)
No Load Current	33.2-30.0/15.0 A
Rated Torque	140 ft.lb
Locked Rotor Torque	300 %
Breakdown Torque	0 %
Insulation Class	F
Service Factor	1.15
Moment of Inertia	11.7 sq.ft.lb
Design	D
Locked Rotor Time	21s (cold) 12s (hot)
Temperature Rise	80 K
Duty Cycle	S1
Maximum Room Temperature	40 °C
Minimum Room Temperature	-20 °C
Degree of Protection	IP55
Altitude	1000 m
Cooling Method	TEFC Totally Enclosed Fan Cooled
Mounting	F-2
Rotation Direction	Both
Noise Level	62.0 dB(A)
Starting Method	-
Enclosure Material	Iron
Front Bearing Shielding	C3
Front Bearing Size	6312
Front Bearing Type	Ball
Front Sealing Type	V Ring
Rear Bearing Shielding	C3
Rear Bearing Size	6212
Rear Bearing Type	Ball
Rear Sealing Type	No Rear Sealing
Grease Type	Mobil Polyrex EM

A1 - Data Specifications for selected AC Induction Motor

A2 - Enclosed Gear Box Isometric View – Original Design



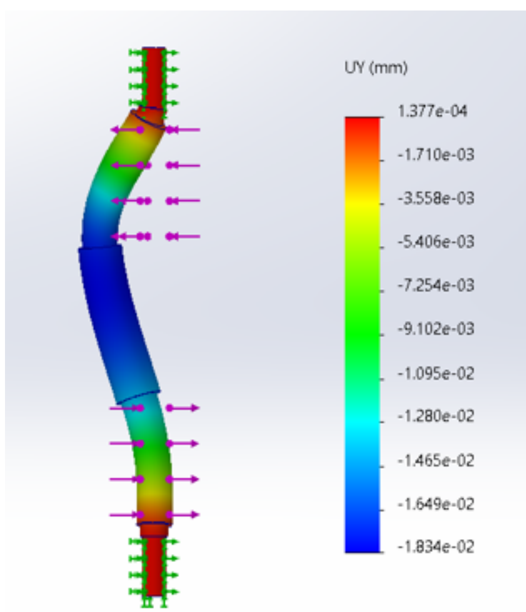
A3 - Exploded Gear View without Housing – Original Design



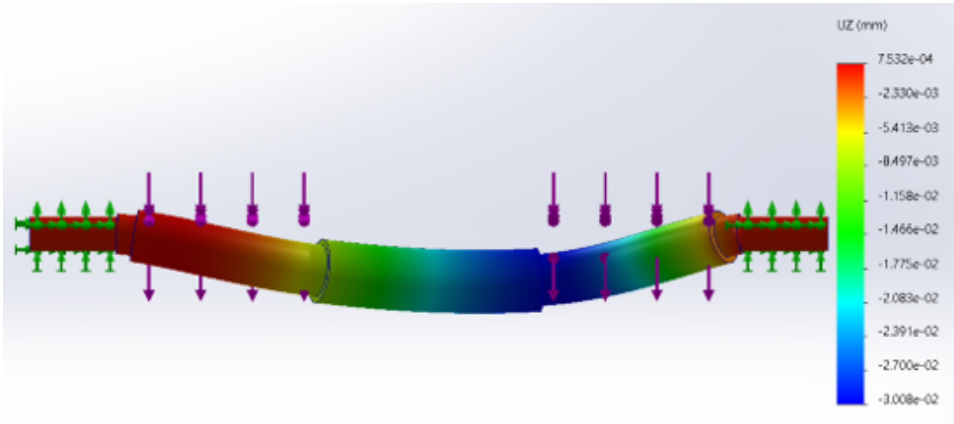
A4 - Total Moment Diagram

Tensile Strength (MPA)	Yield Strength (MPa)	Elongation (%)	Reduction in Area (%)	Brinell Hardness
470	390	15	40	131

Table 3 – Properties of AISI 1020 CD Steel as found in Reference [1]



A5 - Deflection Results in the X-Z Plane 1



A6 - Deflection Results in the X-Y Plane 1

A7 – Final Gearbox Design